****Name: \_\_\_\_SOLUTIONS\_\_\_\_\_\_\_\_\_\_

Full Test (Sections 1 and 2)

Total Time: 58 minutes

Total Marks: 50 marks

Student Result \_\_\_\_\_\_\_\_/ 50

**MATHEMATICS METHODS Unit 3**

**TEST 2 -2021**

**Exponential function,**

**Calculus of trigonometric functions,**

**Discrete random variables**

**Calculator Free Section**

Time: 23 minutes

Total Marks: 20 marks

Resources allowed: SCSA Formula Sheet

**Instructions to candidates**

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks**. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.** If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

|  |  |
| --- | --- |
| **Question 1** | **[2, 2 = 4 marks]** |

Determine given that

(a)

✓ ✓ or shows chain rule use

(b) do not simplify your answer.

✓ quotient rule ✓ correct

|  |  |
| --- | --- |
| **Question 2** | **[2 marks]** |

Verify that the table below describes a probability distribution function with reasons.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

✓

for all or outcomes are exhaustive and mutually exclusive ✓

Therefore, a PDF

|  |  |
| --- | --- |
| **Question 3** | **[2, 3, 2 = 7 marks]** |

(a) Determine the following indefinite integrals.

(i) 

✓ ✓

(ii) 

✓

✓ ✓

(b) Evaluate the following definite integral.

✓

or or

✓

|  |  |
| --- | --- |
| **Question 4** | **[2 marks]** |

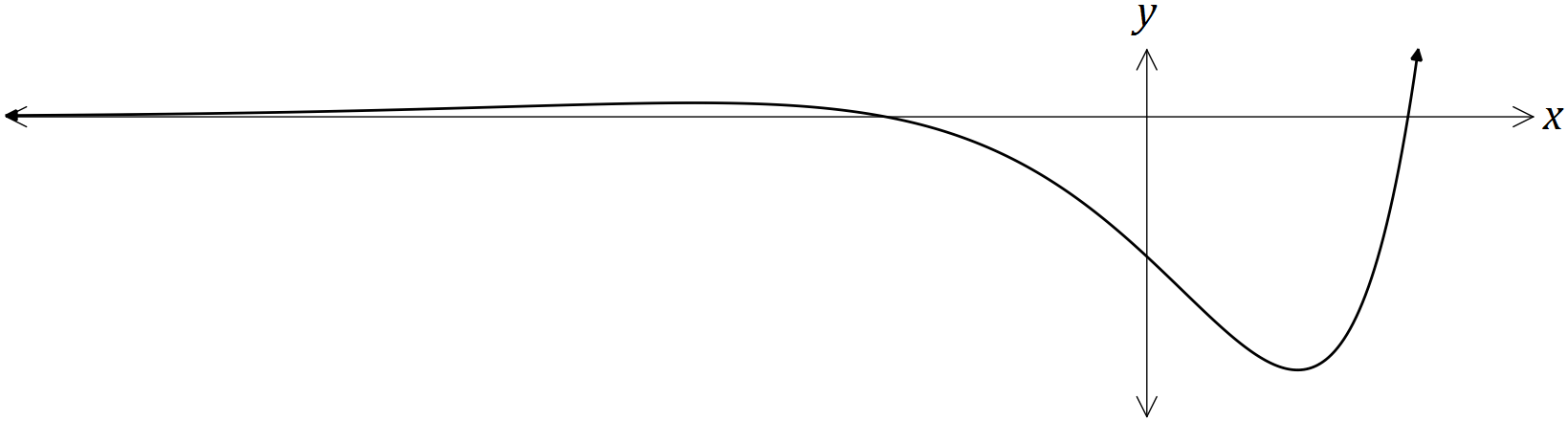
Determine the following. 

✓ uses F.T of C.

✓

|  |  |
| --- | --- |
| **Question 5** | **[2, 3 = 5 marks]** |

The graph of is shown below, where .



(a) Determine , the equation of the gradient function.

✓ uses product rule ✓ correct

(b) Determine the -coordinate only, of all stationary points of .

Solve

for all solve ✓

Stationary points occur when and

✓ ✓

**** Name: \_\_\_\_SOLUTIONS\_\_\_\_\_\_\_\_\_\_

**MATHEMATICS METHODS Unit 3**

**TEST 2 -2021**

**Exponential function,**

**Calculus of trigonometric functions,**

**Discrete random variables**

**Calculator Assumed Section**

Time: 35 minutes

Total Marks: 30 marks

Resources allowed: SCSA Formula Sheet

1 A4 (both side) student notes

CAS calculator

**Instructions to candidates**

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks**. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.** If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

|  |  |
| --- | --- |
| **Question 6** | **[2, 2, 3, 1, 2, 1, 1 = 12 marks]** |

Newton’s Rule of cooling states that the rate of change of the temperature of a particle is proportional to the difference between the temperature of the particle and the constant temperature of the surrounding medium. The temperature, T° C, of a particle when placed in a medium with constant temperature of A° C can be modelled by the equation

T = T0e-kt + A

Where t is the time in minutes and T0 is a constant.

A metal ball has been heated to a temperature of 200°C and is placed into a room that is maintained at a constant temperature of 30°C. After 5 minutes, the temperature of the ball has been dropped to 150°C.

(a) State the value of A and hence show that the value of T0 = 170.

✓

✓

(b) Calculate the value of k correct to 4 decimal places.

✓

CAS solve ✓

(c) Using the values found above, state the equation for this model and sketch its graph

✓

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FXData:
</EFOFEX>

✓ y-int at (0,200) and

asymptote at

✓ exponential decay

(d) Determine the temperature of the rod, correct to 1 decimal place, after 15 minutes.

✓

(e) Calculate the rate of change in temperature, correct to 1 decimal place, at this time.

✓

°C/min ✓

(f) Determine how long it will take for the temperature of the ball to reach 40°C. Give your answer to 2 decimal places.

CAS solve

minutes ✓

(g) Justify why that the metal ball would never reach 10°C if left in the room.

The temperature of the room is 30°C, so the metal ball will approach this temperature and not lower.

Graph is asymptotic at

as

so as ✓ any one of these reasons

|  |  |
| --- | --- |
| **Question 7** | **[1, 3, 1, 3 = 8 marks]** |

The temperature on a particular day can be modelled by the function

,

Where is the time in hours after 5.00 am and is the temperature in degrees Celsius. For the remaining 6 hours of the 24-hour period, the temperature remains constant.

(a) Calculate the temperature at 8.00 am.

✓

(b) At what time(s) of the day is the temperature 20°C? give your answer correct to the nearest minute.

CAS solve

hours hours ✓ values for

11:36 am and 4:25 pm ✓✓ times to the nearest minute

(c) Find

✓

(d) What is the rate of change of temperature at the time(s) found in part (b), correct to two decimal places?

At 11:36 am ✓ correct

At 4:25 pm ✓ correct

✓ correct units used

|  |  |
| --- | --- |
| **Question 8** | **[3 marks]** |

The random variable has mean 40 and standard deviation 3. The random variable

Determine and , where , if the mean and standard deviation of are 110 and 6 respectively.

With

change of scale and origin

… (1)

change of scale only ✓ correct change or scale / origin on mean and S.D.

✓ correct

and from (1)

✓ correct

|  |  |
| --- | --- |
| **Question 9** | **[2, 1, 2, 2 = 7 marks]** |

It is known that 52% of the population participates in sport on a regular basis. Five random individuals are interviewed and asked whether they participate in sport on a regular basis. Let be the number of people who regularly participate in sport.

(a) Complete the probability distribution table for , correct to 4 decimal places.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
|  | 0.0255 | 0.1380 | 0.2990 | 0.3240 | 0.1755 | 0.0380 |

✓ ✓

(b) Determine the



✓

(c) Determine the probability that at least one person plays sport, given that no more than 3 people play sport.

✓ indicates conditional probability

✓

Forty samples of 10 people each from a different country or population were interviewed regarding whether they participate in sport on a regular basis. The following results were recorded, where is the number of people who regularly participate in sport.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| The number of people each the sample that participate in sport on a regular basis, . | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Number of samples with, . | 0 | 0 | 0 | 1 | 3 | 6 | 10 | 10 | 7 | 3 | 0 |

(d) Comment, with justification, the likelihood that this new population has a 52% participation rate in regular sports activities.

The distribution of the discrete random variable Y is symmetrically distributed about approximately 6.5, this suggests that the proportion of people participating in regular sports in this new population is closer to 65%.

Checking: With

From table this is close to Binomial model

✓ suggests a new proportion around 65% ✓ justifies with at least one binomial model check or other reasonable justification.